MathLinks

NUMBERS in BASE TEN 3
STUDENT PACKET

DECIMAL OPERATIONS

Parent (or Guardian) signature _____

MY WORD BANK

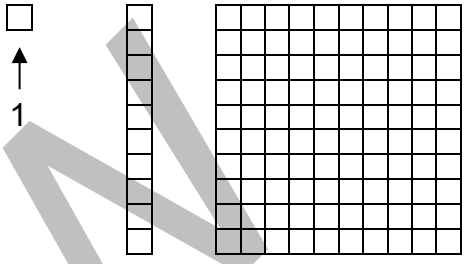
Explain the mathematical meaning of each word or phrase, using pictures and examples when possible. (See section 3.5.) Key mathematical vocabulary is underlined throughout the packet.

Addition, Addend, Sum	Subtraction, Minuend, Subtrahend, Difference
Multiplication, Factor, Product	Division, Dividend, Divisor, Quotient

ONES, TENTHS, AND HUNDREDTHS

Here are pictures of the common base-10 blocks: the “small square,” “the stick,” and the “big square.”

1. Let the small square have a value of 1.



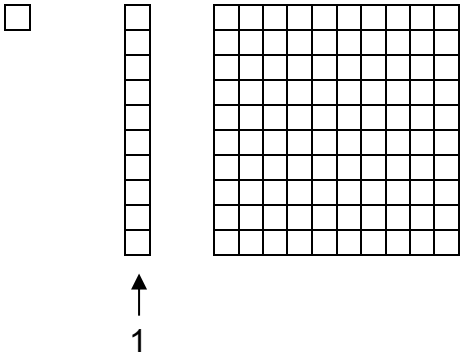
   a. The value of the stick is _____.

   b. The value of the big square is _____.

   c. Shade the big square with 3 tens, 2 tens, 4 ones, and 5 ones.

   d. Write an equation to represent the total from part c.

2. Let the stick have a value of 1.



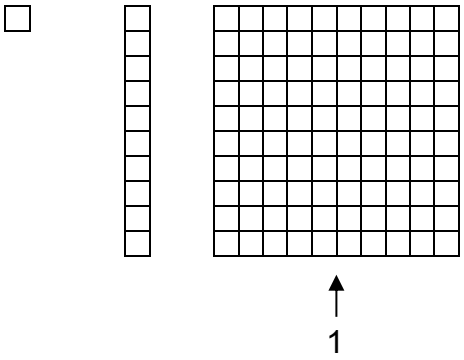
   a. The value of the small square is _____.

   b. The value of the big square is _____.

   c. Shade the big square with 3 ones, 2 ones, 4 tenths, and 5 tenths.

   d. Write an equation to represent the total from part c.

3. Let the big square have a value of 1.



   a. The value of the stick is _____.

   b. The value of the small square is _____.

   c. Shade the big square with 3 tenths, 2 tenths, 4 hundredths, and 5 hundredths.

   d. Write an equation to represent the total from part c.

A CHECKING ACCOUNT

We will add and subtract decimal numbers. We will write checks and keep track of money in a check register.

GETTING STARTED

Write each number in standard form.

1. 8 + 0.06 + 0.005	2. 200 + 3 + 0.09

Write each dollar amount in words.

3. $104	4. $693.07
5. $2,037.64	6. $100.07

7. Circle all numbers that are equivalent to 2.5.

2.50 2.05 25.0 2.5000 250.0 2.500 2.005

8. Explain why 125 and 125.0 are equivalent.

Compute.

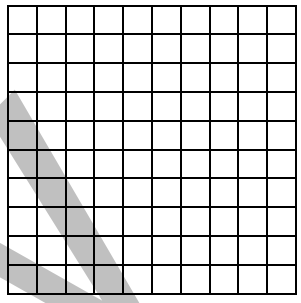
9. 904 + 57	10. 1,078 + 30,456
11. 345 – 21	12. 1,045 – 309

13. Look up addition and subtraction in section 3.5. Write the meanings of all the words and examples related to these operations in My Word Bank.

ADDING AND SUBTRACTING DECIMALS

Follow your teacher's directions.

(1) (2) (3)



(4)	(5)
(6)	(7)
(8)	(9)
(10)	(11)
(12)	(13)

PRACTICE 1

Compute.

1. 129.6 + 7.58	2. 7.456 + 0.67	3. 88.3 + 29.6
4. 4.56 + 1.097	5. 234 + 79.2	6. 0.051 – 0.028
7. 52.17 – 4.6	8. 0.672 – 0.19	9. 827 – 58.2

10. Luke added together the lengths of two videos. One was 25.23 seconds and one was 30.1 seconds. His work is below. Is he correct? Explain.

$$\begin{array}{r} 25.23 \\ +30.1 \\ \hline 282.4 \end{array}$$

11. Samson bought a shirt for $13.50, a sticker for $1.85, and a hat for $23. How much money did he spend altogether?

12. Mariah had $23 on her gift card. She bought a coffee for $4.65. How much money was left on the gift card?

A CHECKING ACCOUNT

Follow your teacher's directions.

(1)

Charlie Stern
10000 W. Wilshire Blvd
Angel City, CA 90024

Date _____ 136

Pay to the
Order of _____ $ [ ]

_____ dollars

First Bank of AC
For _____ _____

(2)

(3)

Check Register

Check Number	Date	Description of Transaction	Debit (–)	Credit (+)	Balance	
		Beginning balance			$ 654	33
135	March 6	TV Superstore	244 50			(2)
136	March 7	Stop 'n Shop	18 86			(2)
	March 12	Deposit – paycheck		566 20		(2)
						(3)
						(3)
						(3)
						(4)

(4)

Charlie Stern
10000 W. Wilshire Blvd
Angel City, CA 90024

Date _____ 139

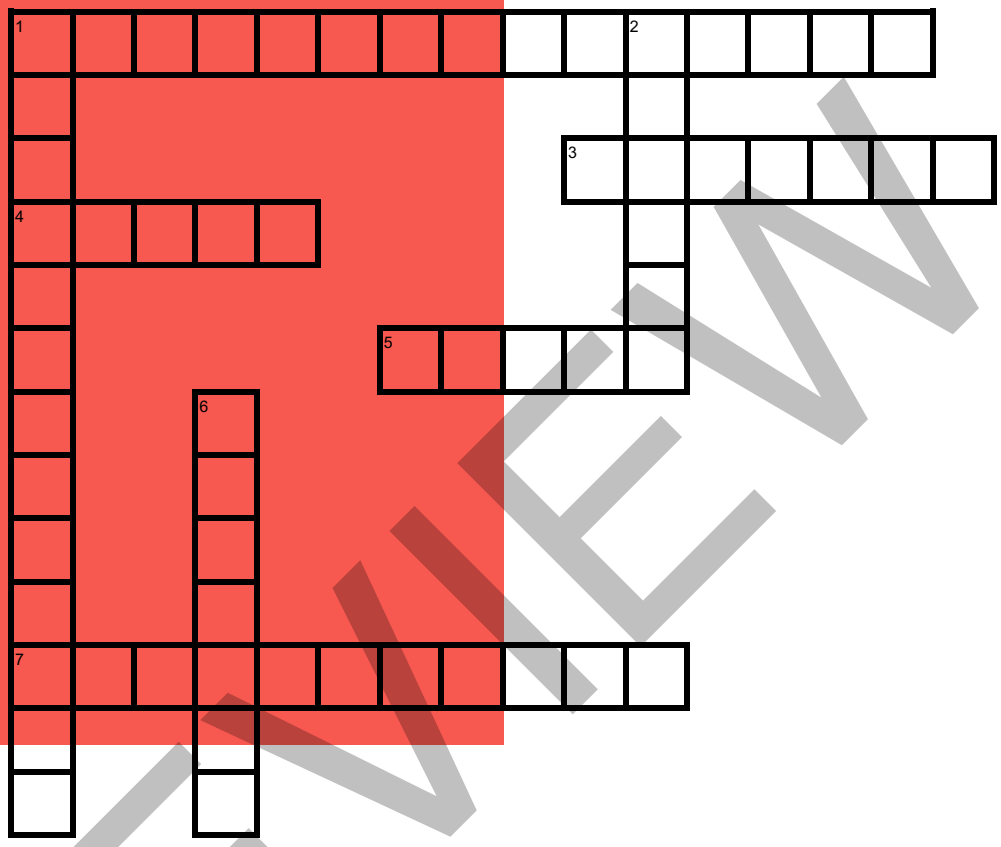
Pay to the
Order of _____ $ [ ]

_____ dollars

First Bank of AC
For _____ _____

PRACTICE 2

Fill in the crossword puzzle using the clues below.



ACROSS

1 A bank account that allows for writing checks (two words).

3 To put money into an account

4 A document that orders a bank to pay the amount specified from an account

5 A withdrawal, recorded as a subtraction

7 Depositing money or writing a check

DOWN

1 A place to keep track of checking account transactions (two words).

2 A deposit, recorded as an addition

6 The current amount of money in an account

Word List		
deposit	check	balance
check register	credit	debit
transaction	checking account	

PRACTICE 3

1. Pretend that you completed the following transactions last year. Record the three checks and the deposit, and keep track of your balance.

   - Check #833 on October 5 to Corner Grocery Store for $18.45
   - Check #834 on October 10 to Party Store for $82.75
   - Check #835 on October 20 to Economy Oil for $40.10
   - Deposit on October 25 for $255

Check Register					
Check Number	Date	Description of Transaction	Debit (–)	Credit (+)	Balance
		Beginning balance			$302 10
833					

2. Write check #836 on Halloween for new cool electronic stuff to Super Electronics for $105.00.

_____ Date _____ 136
4321 Pythagoras Lane
Hypotenuse, USA 31415

Pay to the
Order of _____ $ _____

_____ dollars

Swiss Credit Union
1100 Euler Street
Baseltown, USA 57721

First Bank of AC
For _____ _____

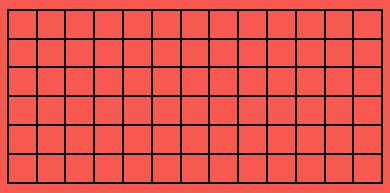
3. What is a good way to remember that a credit is an addition to the balance?

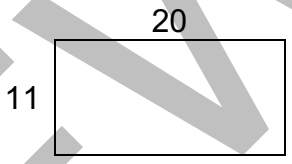
DECIMAL MULTIPLICATION

We will use repeated addition, an area model, and pattern observation to make sense of the rule for multiplying decimal numbers.

GETTING STARTED

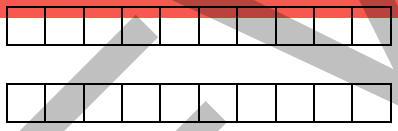
1. Find the area of each rectangle. For part a, each small square is one square unit of area.

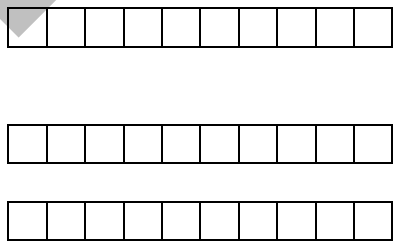
   a. 

   b. 

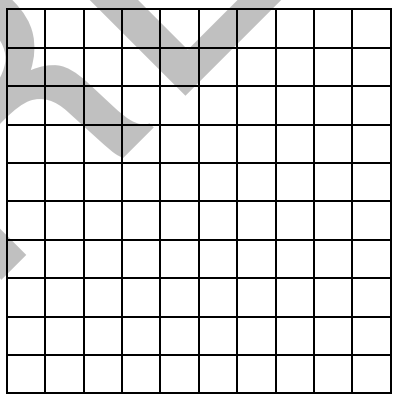
2. Let each stick have a value equal to one. Shade to represent the given value.

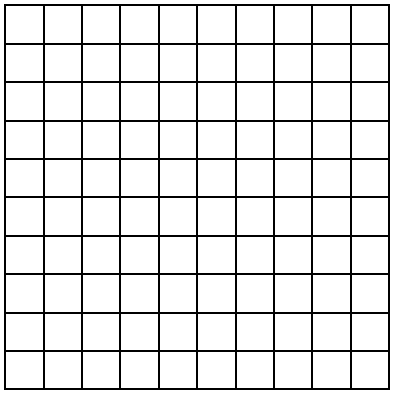
   a. 0.7 

   b. 1.3 

   c. 2.1 

3. Let each big square have a value equal to one. Shade to represent the given value.

   a. 0.21 

   b. 0.54 

4. Compute each sum, mentally if possible.

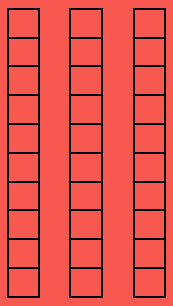
   a. 0.4 + 0.4 b. 0.5 + 0.5 + 0.5 c. 1.3 + 1.3 + 1.3

MULTIPLYING DECIMALS WITH PICTURES

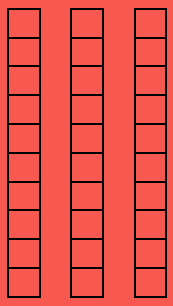
Follow your teacher's directions.

Repeated Addition Model

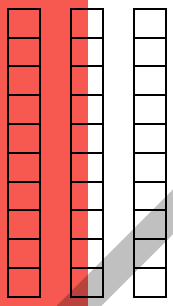
(1)



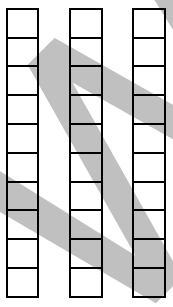
(2)



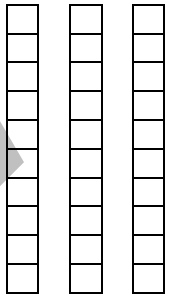
(3)



(4)



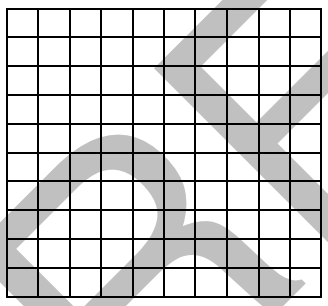
(5)



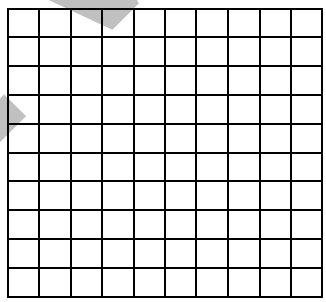
(6)

Area Model

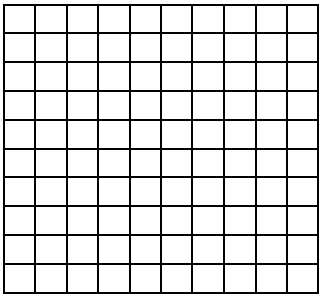
(7)



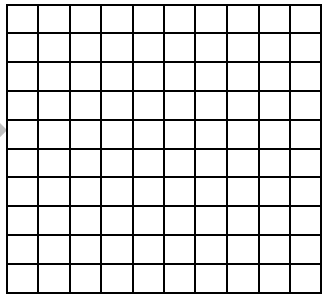
(8)



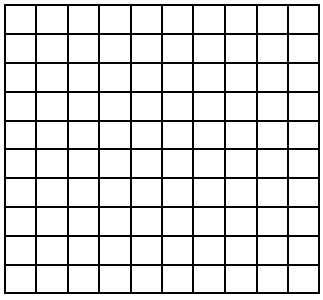
(9)



(10)



(11)

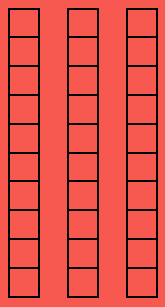


(12)

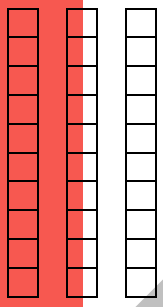
PRACTICE 4

Use repeated addition with a diagram to find each product.

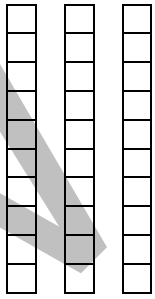
1. 2(0.4)



2. (3)(0.5)

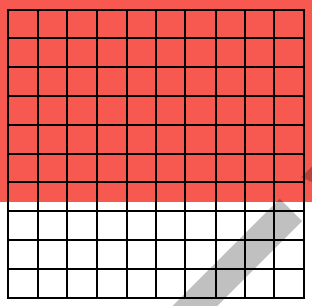


3. 2×1.3

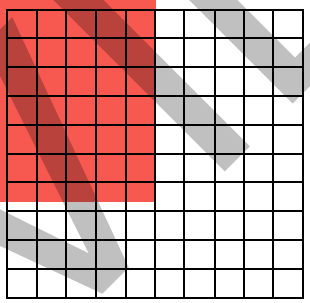


Use an area model to find each product.

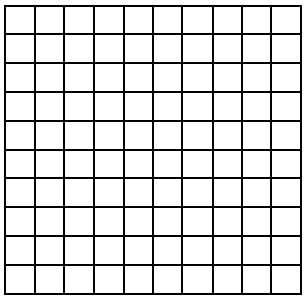
4. (0.2)(0.4)



5. 0.6×0.3



6. (0.5)(0.7)



Use either repeated addition, an area model, or another method of your choice to find each product. If done mentally, explain your process in a few words.

7. 0.9×0.6	8. (1.5)(2)	9. 3(1.6)

10. Look up multiplication in section 3.5. Write the meanings of all the words and examples related to this operation in My Word Bank.

MULTIPLYING DECIMALS WITHOUT PICTURES

Follow your teacher's directions.

(1)	(2)
(3)	(4)

Compute.

5. 3.91×0.3	6. $2.75(0.06)$
7. $(1.005)(2.08)$	8. 0.014×0.96
9. $(8.85)(0.52)$	10. $(1.062)(0.74)$

PRACTICE 5

Multiply mentally.

1. 0.4×0.2	2. 0.04×0.2	3. 0.4×0.02	4. 0.004×0.2

Compute.

5. (0.41)(0.23)	6. (4.1)(0.23)	7. (0.41)(2.3)
8. 34.6 + 8.12	9. 9.1(0.55)	10. 24.6 – 2.8
11. 0.111 + 9.99	12. 1.2(1.2)	13. 2.8 – 0.44

14. Circle the words that best describe the rule for multiplying decimals.

To multiply decimals vertically, line up the numbers on the **right / left**.

Aligning the decimal points is **required** / **not required.**

The number of digits to the **right / left** of the decimal point in the product is equal to the **sum / product** of the number of digits to the right of the decimal point in each factor.

15. Luonda's multiplication is shown to the right. What's her mistake?

$$\begin{array}{r} 1.1 \\ \times\ 0.5 \\ \hline 5.5 \end{array}$$

DECIMAL DIVISION

We will make sense of the rule for dividing decimal numbers.

GETTING STARTED

Use any strategy (numbers, pictures, etc.) to find the following.

1. Four friends share $3.12 so that each one gets the same amount. How much will each friend get?

2. Four friends share $3 so that each one gets the same amount. How much will each friend get?

3. Why are the following numbers equivalent? 3 3.0 3.00

4. Write this division statement in three different ways: $\frac{3}{4} = 0.75$

_____ divided by _____ is _____ _____ ÷ _____ = _____ $\overline{)\ }$

5. Look up division in section 3.5. Write the meanings of all the words and examples related to this operation in My Word Bank.

6. In the division statement $4\overline{)3}\ \ 0.75$, what is the…

dividend? _____ divisor? _____ quotient? _____

QUOTIENTS THAT INVOLVE DECIMALS

Follow your teacher's directions.

(1)	(2)
(3)	(4)
(5)	(6)
(7)	

PRACTICE 6

1. Circle the numbers that are equivalent to 14.3.

   14.03 104.3 140.3 14.30 14.300 14.3000

   Choose one of the circled numbers above and explain how you know it is equivalent to 14.3.

2. Write the division statement in three different ways: 7 divided by 20 equals 0.35

3. Five friends go to lunch and share the cost equally. If the lunch bill is $31.30, how much will each friend pay?

4. Write $\frac{3}{20}$ as a decimal.

   $\frac{3}{20}\left(\frac{5}{5}\right) = \frac{\square}{100} = \_\_\_\_$

   Verify the result with division.

   $20\overline{)\ 3}$

5. Write $\frac{5}{8}$ as a decimal.

   $\frac{5}{8}\left(\frac{\square}{\square}\right) = \frac{\square}{1000} = \_\_\_\_$

   Verify the result with division.

6. Write $\frac{9}{20}$ as a decimal.

7. Write $\frac{7}{8}$ as a decimal.

DIVIDING BY A DECIMAL

Follow your teacher's directions.

(1)	(2)	(3)
(4)	(5)	(6)
(7)	(8)	(9)

10. Write a word problem that would require dividing 7.5 by 1.25.

PRACTICE 7

1. Write this division statement in three different ways: $\frac{1.8}{0.06} = 30$

2. Circle the numbers that are equivalent to 1.05.

   1.50 1.005 1.050 1.0500 01.05 1.5

   Choose one of the numbers not circled above and explain how you know it is not equivalent to 1.05.

3. Explain why $0.08\overline{)3.2}$ is not equivalent to $8\overline{)3200}$.

4. Compute. $\frac{301.53}{0.08}$

5. Compute. $6.48 \div 1.8$

6. Use division to find how many nickels are in $8.75.

7. Dee Harmon thinks that division "makes things smaller." Is she correct? Explain.

BE REASONABLE

Use estimation to choose the most reasonable results for each problem **without computing**.

Circle the most reasonable sum.

1. 8 + 5.3 5.11 6.1 13.3	2. 7.61 + 2.8 7.89 9.69 10.41
3. 0.4 + 0.33 0.37 0.73 4.33	4. 25.5 + 1.11 10.66 26.61 36.6

Circle the most reasonable difference.

5. 8 – 5.3 2.7 3.3 5.5	6. 7.61 – 2.8 4.81 5.53 6.83
7. 0.4 – 0.33 0.07 0.13 0.31	8. 25.5 – 1.11 1.44 14.4 24.39

Circle the most reasonable product.

9. 21.2 × 5.3 11.236 112.36 1123.6	10. 7.62(2) 1.524 15.24 152.4
11. 0.4 × 20 0.8 8 80	12. 0.03(0.6) 0.018 0.18 1.8

Circle the most reasonable quotient.

13. 21.2 ÷ 5.3 0.04 0.4 4	14. 7.62 ÷ 2 0.381 3.81 38.1
15. 0.4 ÷ 20 0.02 5 50	16. 0.03 ÷ 0.6 0.05 0.2 20

17. Check your answers above using a calculator.

PRACTICE 8

1. Freddie's Fruit Stand sells apricots for $0.25, plums for $0.40, and bananas for $0.50 (tax included). List all the ways to spend exactly $2.00 on these fruits.

2. Sarah has $14.56 to spend on beads. She buys 5 bags of large beads that cost $1.15 per bag. She buys 7 bags of medium beads that cost $0.85 per bag.

   a. How much does she have left over to spend on bags of small beads?

   b. If each bag of small beads costs $0.70, how many bags of small beads can she buy?

   c. How much money does she have left over after buying all her beads?

3. Li wants to buy pies for his party. About 30 to 40 people will attend. A pie will serve 4 to 5 people. Each pie costs $8.75. Li's budget is $85. How many pies should Lee buy? Show work and justify your answer.

4. Stanley has a wall in his house that is 11.5 meters long. He wants to hang three pictures equally spaced along the length of the wall. Each picture is 1.4 m wide. He wants to leave 2.1 m space at the left edge and also the right edge. How much space will be between the pictures?

A TARGET GAME

Mrs. Lee played a target game with her class. She removed all of the face cards and tens from a deck of cards. Then, she turned over five cards (2, 7, 8, 2, 1) and called them **decimal cards.** She said, "Let's turn these cards into tenths, so their values will be 0.2, 0.7, 0.8, 0.2, and 0.1." Then she turned over a last card, a 4, and called it the "**target number."** Finally, she challenged the class to make an expression with the decimal cards that was as close to the target number as possible, using any mathematical operations or symbols they wanted.

1. Here are some expressions two students created. Find their values. Who won the game?

Jayme's expression:	Damond's expression:
0.2 + 0.7 + 0.8 + 0.2 + 0.1	(0.8 ÷ 0.2)(0.2 + 0.7 + 0.1)

2. Using your own paper and a deck of cards, play several rounds of the target game with your class or a partner.

Record your two best rounds here.

_____ _____ _____ _____ _____ _____

(decimal cards) (target number)

My expression and its value:

_____ _____ _____ _____ _____ _____

(decimal cards) (target number)

My expression and its value:

REVIEW

POSTER PROBLEM

Part 1: Your teacher will divide you into groups.

- Identify members of your group as A, B, C, or D.
- Each group will start at a numbered poster. Our group start poster is _____.
- Each group will have a different colored marker. Our group marker is _____.

Part 2: Do the problems on the posters by following your teacher's directions.

	Poster 1 (or 5)	Poster 2 (or 6)	Poster 3 (or 7)	Poster 4 (or 8)
M	143.6	59.22	21.07	207.01
N	37.15	7.8	16.45	12.9
P	17	9.2	96	9.8
Q	0.02	0.04	1.2	0.14

A. Add M and N. Show all work.

B. Subtract N from M. Show all work.

C. Multiply P and Q. Show all work.

D. Divide P by Q. Show all work.

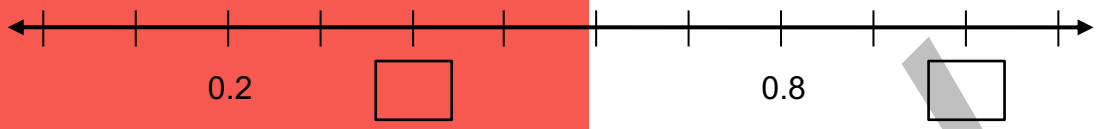
Part 3: Return to your start poster.

- Choose any problem on your start poster and write a word problem that translates into doing this problem.

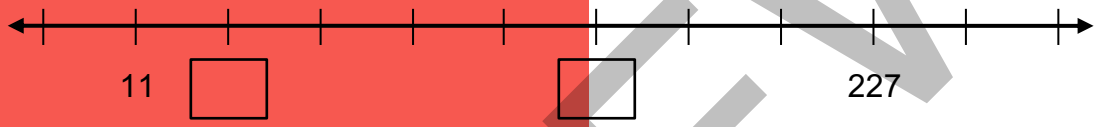
FIND THE MISSING VALUES

All tick marks are equally spaced on each number line below. Find the unknown values for each number line. Explain your reasoning or show your calculations.

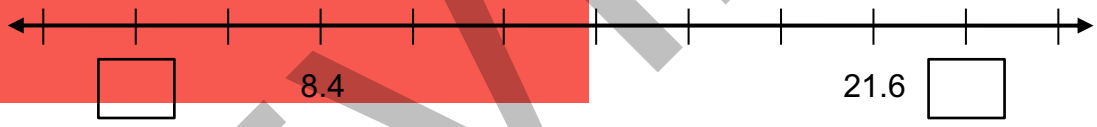
1.



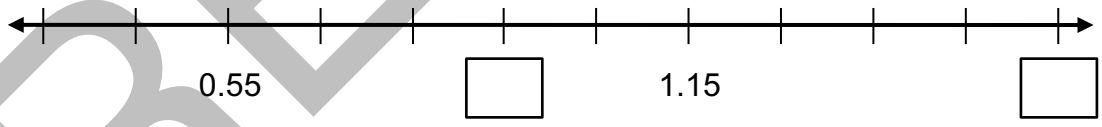
2.



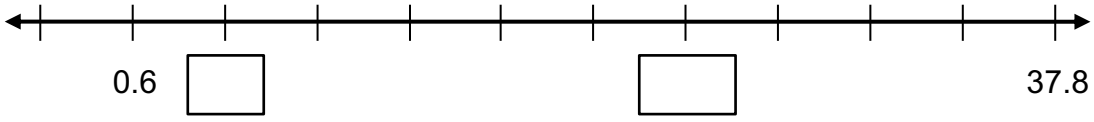
3.



4.



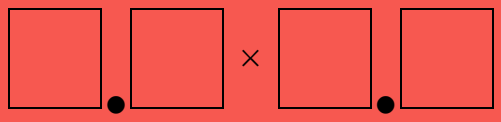
5.



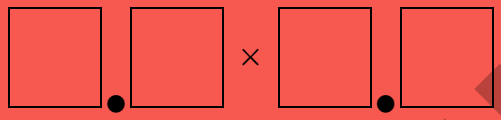
MULTIPLICATION AND DIVISION TARGETS

Use the digits 1-9 no more than once each for each problem.

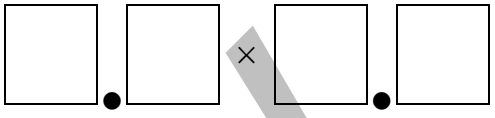
1a. Make a product for:



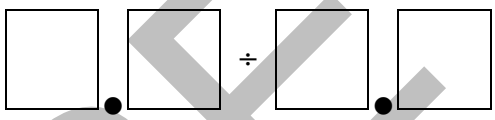
1b. Make the greatest product for:



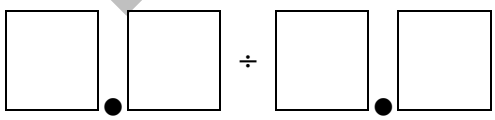
2. Make the product closest to zero for:



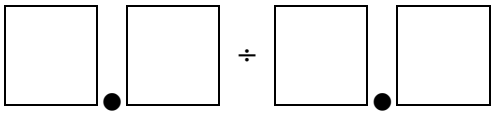
3a. Make a quotient for:



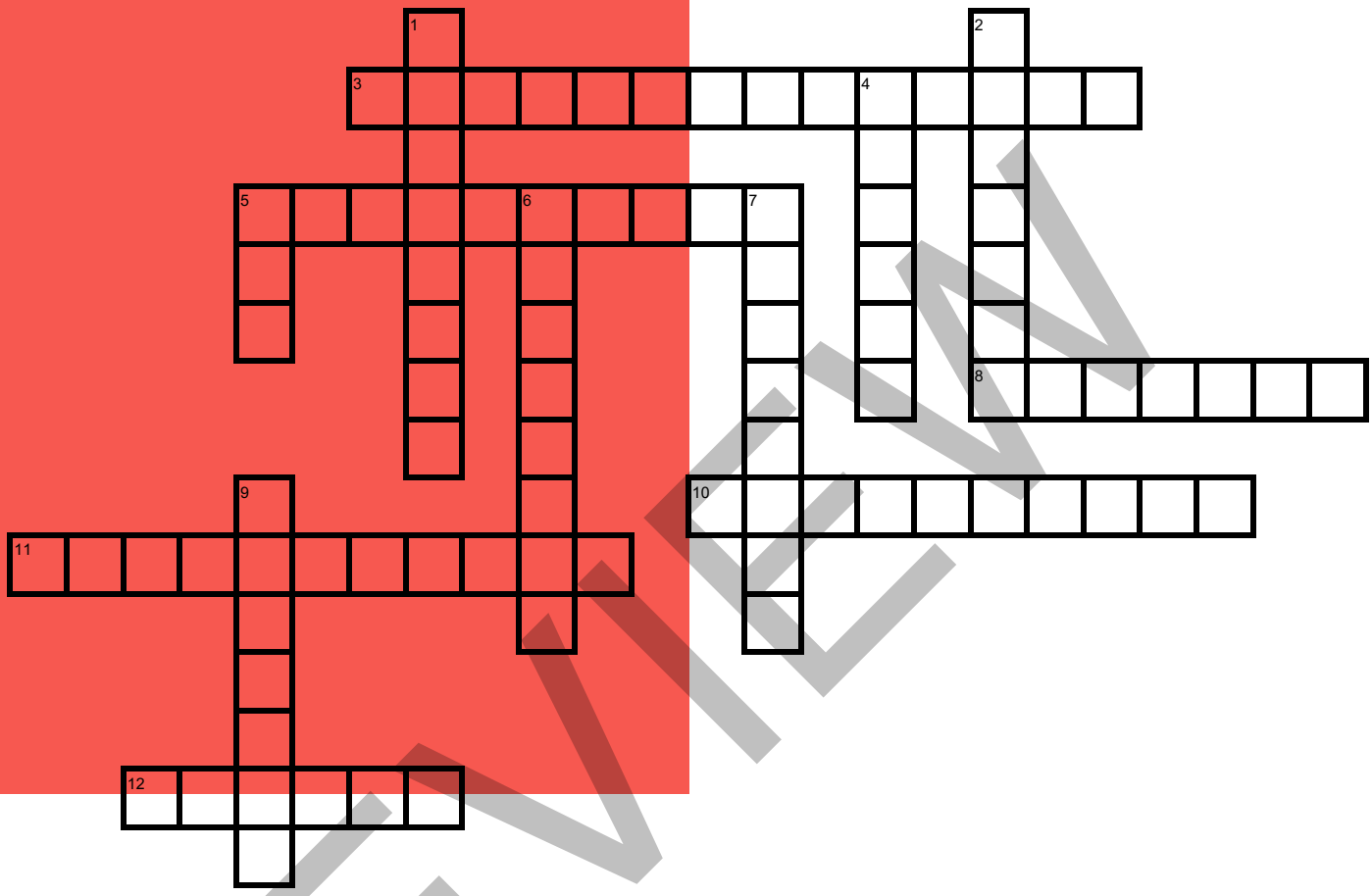
3b. Make the greatest quotient for:



4. Make the quotient closest to zero for:



VOCABULARY REVIEW



Across

3 An operation that counts “groups of”

5 Name of this part of the equation:
25 – 4 = 21

8 Name of this part of the equation:
825 ÷ 25 = 33

10 The result of subtraction

11 Operation illustrated in 2 down

12 Name of this part of the equation:
(35)(3) = 105

Down

1 The result of division

2 Name of this part of the equation:
25 – 4 = 21

4 Name of this part of the equation:
37 + 14 = 41

5 The result of addition

6 Operation that combines numbers

7 Operation that makes “groups of”

9 The result of multiplication.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES

Word or Phrase	Definition
addition	In an addition problem, the sum is the result of addition. The numbers to be added to form the sum are addends. 7 + 5 = 12 addend addend sum
algorithm	An algorithm is an organized procedure, or step-by-step recipe, for performing a calculation or finding a solution. The traditional procedure for dividing whole numbers is called the long division algorithm.
conjecture	A conjecture is a statement that is proposed to be true, but has neither been proven to be true nor to be false.
division	Division is the mathematical operation that is inverse to multiplication. For $b \neq 0$, division by b is multiplication by the multiplicative inverse $\frac{1}{b}$ of b, $a \div b = a \bullet \frac{1}{b}$. In this division problem, the number a to be divided is the dividend, the number b by which a is divided is the divisor, and the result $a \div b$ of the division is the quotient: $\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$ $\qquad$ $\text{divisor}\overline{)\text{dividend}}^{\text{quotient}}$ Twelve divided by 2 may be written $12 \div 2$, $\frac{12}{2}$, or $2\overline{)12}$. 12 ÷ 2 = 6 dividend divisor quotient
multiplication	In a multiplication problem, the product is the result of multiplication of two or more numbers or expressions. The numbers or expressions being multiplied to form the product are factors of the product. 7 • 8 = 56 factor factor product
subtraction	In a subtraction problem, the difference is the result of subtraction. The minuend is the number from which another number is being subtracted, and the subtrahend is the number that is being subtracted. 12 – 4 = 8 minuend subtrahend difference

Notation for Multiplication

The product of 8 and 4 can be written as:

8 times 4	8×4	$8 \bullet 4$	(8)(4)	8(4)	$\begin{array}{r} 8 \\ \times\ 4 \end{array}$

In algebra, we generally avoid using the $\times$ for multiplication because it could be misinterpreted as the variable *x*, and we cautiously use the symbol $\bullet$ for multiplication because it could be misinterpreted as a decimal point.

Notation for Division

The quotient of 8 and 4 can be written as:

8 divided by 4	$8 \div 4$	$4\overline{)8}$	$\frac{8}{4}$	8/4

In algebra, the preferred way to show division is with fraction notation.

Standard Algorithms for Adding and Subtracting Decimal Numbers

Addition

- Set up the problem in columns, with place values lined up to add tens with tens, ones with ones, tenths with tenths, etc. When the digits are properly lined up, the decimal points will also align.
- (Optional) Include trailing zeroes to the right of the decimal points as place holders if needed, as in this problem where 1 thousandth is added to 0 thousandths.
- Add with regrouping as usual. Since the place values in the sum line up with the place values in the two addends, the decimal point in the sum will align with the decimal points in the addends.

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ \ \ } \\ 48.560 \\ +\ 36.521 \\ \hline 85.081 \end{array}$$

Subtraction

- Set up the problem in columns, with place values lined up to subtract tens from tens, ones from ones, tenths from tenths, etc. When the digits are properly lined up, the decimal points will also align.
- Include trailing zeroes to the right of the decimal point as place holders in the minuend (top number) as needed to line up with any trailing nonzero digit in the subtrahend (bottom number).
- Subtract as though the decimal points are not there. When done calculating, place the decimal point in the difference directly below the decimal points in the problem.

$$\begin{array}{r} {\scriptstyle 6\ \ 13\ 10\ } \\ 7.40 \\ -\ 3.51 \\ \hline 3.89 \end{array}$$

Multiplying Decimal Numbers

Repeated Addition

Example 1: 3×0.4

Each strip represents one whole. Each part represents 0.1.
Shade 0.4, a second 0.4, and then a third 0.4. for a total of 1.2.

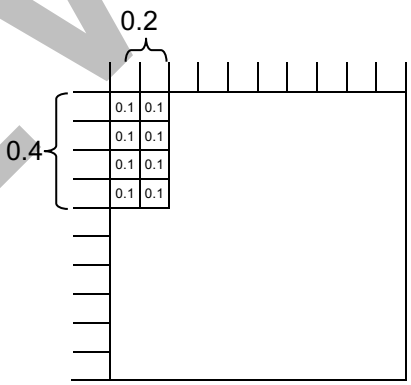
$$\begin{array}{r} 0.4 \\ 0.4 \\ +\ 0.4 \\ \hline 1.2 \end{array}$$

Area Model

Example 2: 0.2×0.4

As with whole numbers and fractions, an area model can be used for decimal multiplication. Start with a unit square (a 1 x 1 square with an area of 1 square unit).

The factors (0.2 and 0.4) represent the side lengths of a rectangle, and the product (0.08) is its area.



Fraction Equivalents

Example 3: 0.03×0.2

Since $0.03 = \frac{3}{100}$ and $0.2 = \frac{2}{10}$, $0.03 \times 0.2 = \frac{3}{100} \times \frac{2}{10}$.

Using the fraction multiplication rule: $\frac{3}{100} \times \frac{2}{10} = \frac{6}{1{,}000}$.

Since $\frac{6}{1{,}000} = 0.006$, $0.03 \times 0.2 = 0.006$.

The examples above show that when multiplying decimals, the number of digits to the right of the decimal point in the product is equal to the *sum* of the number of digits to the right of the decimal point of each factor.

$3 \times 0.4 = 1.2$	$0.2 \times 0.4 = 0.08$	$0.03 \times 0.2 = 0.006$
0 digits 1 digit 1 digit	1 digit 1 digit 2 digits	2 digits 1 digit 3 digits

Equivalence and the "Big One"

The multiplication property of 1 states that $a \bullet 1 = 1 \bullet a = a$ for all numbers a. In other words, 1 is a multiplicative identity. We sometime call 1 in a fraction form the "big 1." The "big 1" is a notation for 1 in the form of a fraction $\frac{n}{n}$ $(n \neq 0)$. For example,

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \ldots$$

We can use the following picture to help remind us that these fractions are equivalent to 1:

$$1 = \boxed{\frac{8}{8}}$$

The "big 1" can be used to show equivalence of fractions. For example,

$$\frac{2}{5} \times \boxed{\frac{10}{10}} = \frac{20}{50} \qquad \text{or} \qquad \frac{20}{50} \div \boxed{\frac{10}{10}} = \frac{2}{5}$$

Dividing Decimal Numbers

The procedure for dividing decimals involves "moving the decimal point." The reason this is done is because we usually consider dividing by a whole number to be an easier process.

Consider $12.5 \div 0.25$, which can be written as $0.25\overline{)12.5}$ or $\frac{12.5}{0.25}$.

When $12.5 \div 0.25$ is multiplied by 1 in the form of $\boxed{\frac{100}{100}}$, it is equal to $1250 \div 25$.

That is, $\frac{12.5}{0.25} \bullet \boxed{\frac{100}{100}} = \frac{1250}{25}$.

Now we can divide by a whole number. This process often is depicted this way:

$$0.25\overline{)12.5} \rightarrow 0.25\overline{)12.50} \rightarrow 025.\overline{)1250.} \rightarrow 25.\overline{)1250.}^{\,50.}$$

Standard Algorithms for Multiplying and Dividing Decimal Numbers	
Multiplication • Multiply, ignoring the decimal points. • **Then** put the decimal point in the product. The product will have as many places to the right of the decimal point as the two original factors combined.	3.4 × 4.0 5 1 7 0 + 1 3 6 0 0 1 3. 7 7 0
Division • Multiply the divisor and dividend by the same power of 10 (10, 100, 1000, etc.) so that the divisor is a whole number. • Divide as usual, lining up the digits of the quotient above the dividend so that the tens line up with tens, ones with ones, tenths with tenths, and so on. Place the decimal in the quotient in the same location as the dividend.	0.02) 0.358 17.9 2) 35.8 − 2 15 − 14 18 − 18 0

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